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# Left-Right Gauge Model in Nonassociative Geometry

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**Abstract**

We reformulate the left-right gauge model of Pati-Mohapatra using nonassociative geometry approach. At the tree level we obtain the mass relations $M_W = \frac{1}{2}m_t$, $M_H = \frac{3}{2}m_t$ and the mixing angles are $\sin^2\theta_W = \frac{3}{8}$ and $\sin^2\theta_s = \frac{3}{5}$, which are identical to the ones obtained in $SO(10)$ GUT .

# 1 Introduction

One of the greatest achievement of noncommutative geometry (abbreviated **NCG** hereafter) is its geometrization of the standard model ([?],[?],[?]). NCG provides a framework where the Higgs boson $H$ may be introduced on the same level as $W^{\pm}$ and $Z$ bosons. In this approach we introduce additional discrete dimensions to the four-dimensional space-time. If the gauge bosons are associated to the continuous directions, the Higgs boson results from gauging the discrete directions. However one of the shortcomings of NCG is its non accessibility to grand unified theories GUT ([?],[?]). The other formulation of NCG due to Coquereaux [?] does not suffer from this problem ([?]-[?]). There are some variant of Connes' theory ([?],[?]) where we use an auxiliary Hilbert space to fit GUT.

R. Wulkenhaar has recently succeeded in formulating another type of geometry which shares a lot of common points with NCG à la Connes [?]. The theory was baptized "nonassociative geometry" **NAG**. The main difference with the two theories is that NAG is based on unitary Lie algebra instead of unital associative $\star$-algebra in the case of NCG. Its application to several physical models has been successful ([?]-[?]).

We try using this approach to reformulate the left-right model LRM ([?],[?]). One of the features of NAG (and NCG) is its geometric explanation of the spontaneous breakdown of gauge symmetry. Our aim is to investigate whether NAG could encompass also the parity violation.

In section 2 we present the main elements to formulate gauge theories using NAG. In section 3 we derive the bosonic action of left-right model and finally we discuss our results.

# 2 Some elements of Nonassociative Geometry

NAG is based on the L-cycle $(\mathfrak{g}, \mathcal{H}, D, \pi, \Gamma)$ [?]

$$\mathfrak{g} = C^{\infty}(X) \otimes \mathfrak{a} \tag{1}$$

where $C^{\infty}(X)$ is the algebra of smooth functions on the manifold X and $\mathfrak{a}$ is the matrix Lie algebra. $\mathcal{H}$ is the Hilbert space

$$\mathcal{H} = L^2(X, S) \otimes \mathbb{C}^F \tag{2}$$

where $L^2(X,S)$ is the Hilbert space of spinors.

$D$ is the total Dirac operator given by:

$$D = \mathbf{D} \otimes \mathbf{1}_F + \gamma^5 \otimes \mathcal{M} \tag{3}$$

where $\mathbf{D}$ is the Dirac operator associated with the continuous algebra $C^\infty(X)$ and $\mathcal{M}$ is associated to the discrete algebra $\mathfrak{a}$.

The representation of $\mathfrak{g}$ on $\mathcal{H}$ is given by

$$\pi = \mathbf{1} \otimes \hat{\pi}, \tag{4}$$

where $\hat{\pi}$ is the representation of $\mathfrak{a}$ on $\mathbb{C}^F$.

Finally, the graded operator $\Gamma$, acting on $\mathcal{H}$, is given by:

$$\Gamma = \gamma^5 \otimes \hat{\Gamma}, \tag{5}$$

where $\hat{\Gamma}$ is the grading operator acting on $\mathbb{C}^F$.

The space $\hat{\Omega}^1\mathfrak{a}$ is generated by the elements of the type

$$\omega^1 = \sum_z [a^z, ...[a^1, da^0]...] \qquad a^i \in \mathfrak{a}. \tag{6}$$

The representation $\hat{\pi}$ acts on the space $\hat{\Omega}^1\mathfrak{a}$ as:

$$\hat{\pi} : \hat{\Omega}^1\mathfrak{a} \longrightarrow \mathbb{M}_F(\mathbb{C})$$

$$\tau^1 = \hat{\pi}(\omega^1) := \sum_z [\hat{\pi}(a^z), ...[\hat{\pi}(a^1), [-i\mathcal{M}, \hat{\pi}(a^0)]]...] \tag{7}$$

We define also the mapping:

$$\hat{\sigma} : \hat{\Omega}^1\mathfrak{a} \longrightarrow \mathbb{M}_F(\mathbb{C})$$

$$\hat{\sigma}(\omega^1) := \sum_z [\hat{\pi}(a^z), ...[\hat{\pi}(a^1), [\mathcal{M}^2, \hat{\pi}(a^0)]]...] \tag{8}$$

For n$\geqslant$ 2 we have

$$\pi(J^{k+1}\mathfrak{g}) = \left\{ \sigma(\omega^k); \omega^k \in \hat{\Omega}^k\mathfrak{g} \cap \ker(\pi) \right\}. \tag{9}$$